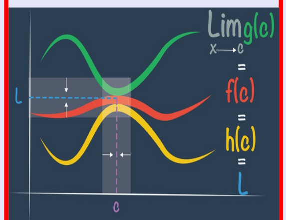
Math 261

Fall 2022

Lecture 2



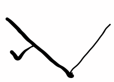
Consider the piece-wise function below

$$f(x) = \begin{cases} |x| & \text{if } x \le 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$$

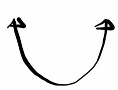
1) $f(-2) = |-2| = 2$

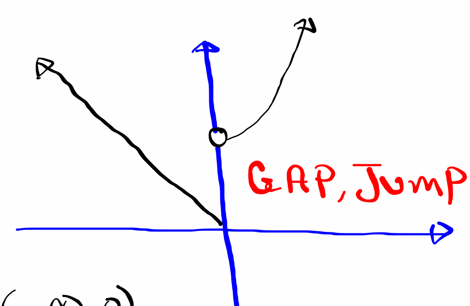
2) $f(2) = 2^2+1 = 5$

$f(x) = |x|$



$f(x) = x^2+1$



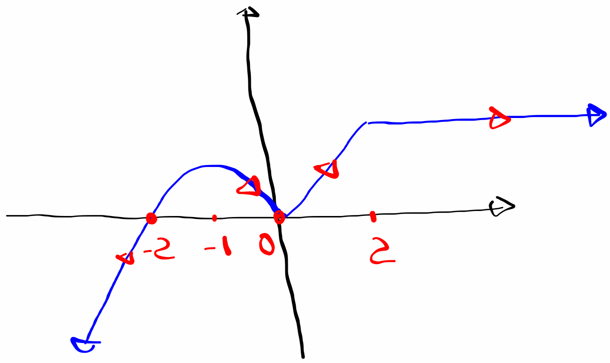


x-Int $(0,0)$
Y-Int $(0,0)$

Decreasing $(-\infty, 0)$

Increasing $(0, \infty)$

Consider the graph below



1) x-Int $(-2,0), (0,0)$

2) Y-Int $(0,0)$

3) Increasing

$(-\infty,-1) \cup (0,2)$

4) Decreasing

$(-1,0)$

5) Constant $(2,\infty)$

Consider $f(x) = x^2 - 4x + 4$

x-Int

$y = 0$

$f(x) = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$

$x = 2$

x-Int $(2,0)$

Y-Int

$x = 0$

$f(0) = 0^2 - 4(0) + 4$

$= 4$

Y-Int $(0,4)$

$f(x) = x^2 - 4x + 4$

Parabola

opens upward

Vertex (h, k)

$h = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$k = f(h) = f(2) = 0$

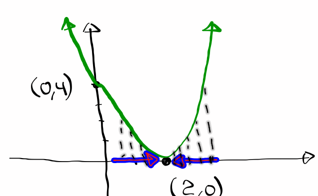
Vertex $(2,0)$

$f(x) = x^2 - 4x + 4$

$= (x-2)^2$

Take $f(x) = x^2$

shift Right 2 units



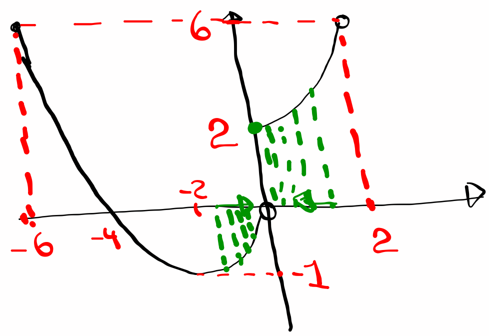
As x approaches 2 from the right

$f(x) \to 0$

As x approaches 2 from the left

$f(x) \to 0$

Consider the graph below



1) Domain $[-6,2)$

2) Range $[-1,6]$

3) X-Int $(-4,0)$

4) Y-Int $(0,2)$

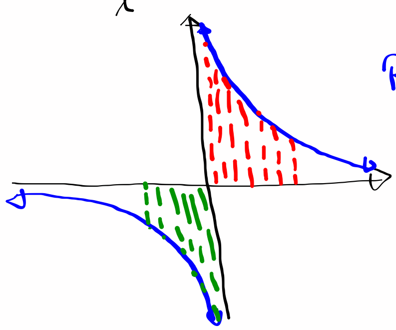
5) Decreasing $(-6,-2)$

6) Increasing $(-2,2)$

7) As $x \to 0$ from the right $f(x) \to 2$

8) As $x \to 0$ from the left $f(x) \to 0$

$f(x) = \frac{1}{x}$



Domain $(-\infty,0) \cup (0,\infty)$

Range $(-\infty,0) \cup (0,\infty)$

As x approaches 0 from Right

$x \to 0^+$ $f(x) \to \infty$

As x approaches 0 from left

$x \to 0^-$ $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to 0$

As $x \to -\infty$, $f(x) \to 0$

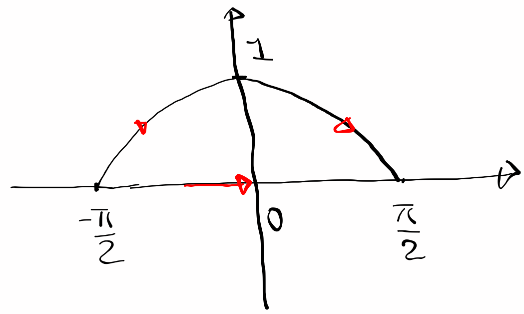
Graph $f(x) = \cos x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: $[0, 1]$

x-Int $\left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right)$

Y-Int $(0, 1)$



Increasing $\left(-\frac{\pi}{2}, 0\right)$ Decreasing $\left(0, \frac{\pi}{2}\right)$

$\underbrace{\text{As } x \to 0^+,\ f(x) \to 1,\ \text{As } x \to 0^-,\ f(x) \to 1}$

Introduction to limits

Difference Quotient for function $f(x)$ $\to$ $\dfrac{f(x+h) - f(x)}{h}$

Find difference quotient for $f(x) = x^2$, evaluate the final answer for $h=0$.

$$\frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h} = \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x + h$$

For $h=0 \Rightarrow \boxed{2x}$

Repeat last example for

$f(x) = \frac{1}{x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$\boxed{LCD = x(x+h)}$

$$= \frac{x\cancel{(x+h)} \cdot \frac{1}{\cancel{x+h}} - \cancel{x}(x+h) \cdot \frac{1}{\cancel{x}}}{x(x+h) \cdot h}$$

$$= \frac{x - (x+h)}{x(x+h)h} = \frac{-\cancel{h}}{x(x+h)\cancel{h}}$$

$$= \frac{-1}{x(x+h)}$$

For $h=0$

$$\frac{-1}{x^2}$$

Repeat last example for $f(x) = \sqrt{x}$

Rationalize the numerator

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Recall

$(A-B)(A+B) = A^2 - B^2$

$$= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\overset{1}{\cancel{x} + \cancel{h} - \cancel{x}}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

For $h=0$

$$\rightarrow \frac{1}{2\sqrt{x}}$$